

# 「論理と意味論」演習問題解答

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演習 3.4 以下の式の冠頭連言標準形を求めよ。ただし、 $x, y, z$  は個体変項であり、 $a, b$  は個体定項とする。また、詳細な変換過程も示すこと。

1.  $\forall x(P(x) \rightarrow \exists yQ(x, y))$

$$\Rightarrow \forall x(\sim P(x) \vee \exists yQ(x, y))$$

$$\Rightarrow (\text{答}) \quad \forall x\exists y(\sim P(x, y) \vee Q(x, y))$$

2.  $\forall x \sim Q(x, x) \rightarrow \forall y \sim P(y)$

$$\Rightarrow \sim \forall x \sim Q(x, x) \vee \forall y \sim P(y)$$

$$\Rightarrow \exists x \sim \sim Q(x, x) \vee \forall y \sim P(y)$$

$$\Rightarrow \exists xQ(x, x) \vee \forall y \sim P(y)$$

$$\Rightarrow (\text{答}) \quad \exists x\forall y(Q(x, x) \vee \sim P(y))$$

3.  $\exists x(P(x) \rightarrow \exists y(Q(x, y) \rightarrow R(x, y)))$

$$\Rightarrow \exists x(\sim P(x) \vee \exists y(\sim Q(x, y) \vee R(x, y)))$$

$$\Rightarrow (\text{答}) \quad \exists x\exists y(\sim P(x) \vee \sim Q(x, y) \vee R(x, y))$$

4.  $\forall x\forall y(P(x, y) \wedge Q(x, y)) \rightarrow \forall x\forall yR(x, y)$

$$\Rightarrow \sim \forall x\forall y(P(x, y) \wedge Q(x, y)) \vee \forall u\forall vR(u, v)$$

$$\Rightarrow \exists x \sim \forall y(P(x, y) \wedge Q(x, y)) \vee \forall u\forall vR(u, v)$$

$$\Rightarrow \exists x\exists y \sim (P(x, y) \wedge Q(x, y)) \vee \forall u\forall vR(u, v)$$

$$\Rightarrow \exists x\exists y(\sim P(x, y) \vee \sim Q(x, y)) \vee \forall u\forall vR(u, v)$$

$$\Rightarrow (\text{答}) \quad \exists x\exists y\forall u\forall v(\sim P(x, y) \vee \sim Q(x, y) \vee R(u, v))$$

5.  $\forall x(P(a, b) \rightarrow \exists y(Q(a, y) \vee R(y, y)))$

$$\Rightarrow (\sim P(a, b) \vee \exists y(Q(a, y) \vee R(y, y)))$$

$$\Rightarrow (\text{答}) \quad \exists y(\sim P(a, b) \vee Q(a, y) \vee R(y, y))$$

6.  $\forall x\forall y(P(x, y) \vee Q(x, y)) \wedge \sim \forall x\forall y(R(x, y) \vee P(x, x))$

$$\Rightarrow \forall x\forall y(P(x, y) \vee Q(x, y)) \wedge \sim \forall u\forall v(R(u, v) \vee P(u, u))$$

$$\Rightarrow \forall x\forall y(P(x, y) \vee Q(x, y)) \wedge \exists u \sim \forall v(R(u, v) \vee P(u, u))$$

$$\Rightarrow \forall x\forall y(P(x, y) \vee Q(x, y)) \wedge \exists u\exists v \sim (R(u, v) \vee P(u, u))$$

$$\Rightarrow \forall x\forall y(P(x, y) \vee Q(x, y)) \wedge \exists u\exists v(\sim R(u, v) \wedge \sim P(u, u))$$

$$\Rightarrow (\text{答}) \quad \exists u\exists v\forall x\forall y((P(x, y) \vee Q(x, y)) \wedge \sim R(u, v) \wedge \sim P(u, u))$$

7.  $\exists x(\sim(\exists yP(x, y) \rightarrow (\exists zQ(z) \rightarrow R(x))))$   
 $\implies \exists x(\sim(\sim \exists yP(x, y) \vee (\sim \exists zQ(z) \vee R(x))))$   
 $\implies \exists x(\sim\sim \exists yP(x, y) \wedge (\sim \forall z \sim Q(z) \vee R(x)))$   
 $\implies \exists x(\exists yP(x, y) \wedge (\sim \forall z \sim Q(z) \wedge \sim R(x)))$   
 $\implies \exists x(\exists yP(x, y) \wedge (\exists z \sim\sim Q(z) \wedge \sim R(x)))$   
 $\implies \exists x(\exists yP(x, y) \wedge (\exists zQ(z) \wedge \sim R(x)))$   
 $\implies$  (答)  $\exists x\exists y\exists z(P(x, y) \wedge Q(z) \wedge \sim R(x))$
8.  $\forall x\forall y(\exists zP(x, y, z) \wedge (\exists uQ(x, u) \rightarrow \exists vQ(y, v)))$   
 $\implies \forall x\forall y(\exists zP(x, y, z) \wedge (\sim \exists uQ(x, u) \vee \exists vQ(y, v)))$   
 $\implies \forall x\forall y(\exists zP(x, y, z) \wedge (\forall u \sim Q(x, u) \vee \exists vQ(y, v)))$   
 $\implies$  (答)  $\forall x\forall y\exists z\exists v\forall u(P(x, y, z) \wedge (\sim Q(x, u) \vee Q(y, v)))$
9.  $\sim \forall x\{(P(x) \wedge \exists y[Q(x, y) \vee R(y)]) \rightarrow \exists zS(x, z)\}$   
 $\implies \sim \forall x\{\sim (P(x) \wedge \exists y[Q(x, y) \vee R(y)]) \vee \exists zS(x, z)\}$   
 $\implies \exists x \sim \{\sim (P(x) \wedge \exists y[Q(x, y) \vee R(y)]) \vee \exists zS(x, z)\}$   
 $\implies \exists x\{(P(x) \wedge \exists y[Q(x, y) \vee R(y)]) \wedge \sim \exists zS(x, z)\}$   
 $\implies \exists x\{(P(x) \wedge \exists y[Q(x, y) \vee R(y)]) \wedge \forall z \sim S(x, z)\}$   
 $\implies$  (答)  $\exists x\exists y\forall z\{P(x) \wedge (Q(x, y) \vee R(y)) \wedge \sim S(x, z)\}$
10.  $\sim \forall x\{P(x) \vee \exists y[(Q(x, y) \rightarrow Q(y, x)) \vee \forall z(Q(y, z) \rightarrow R(x))]\}$   
 $\implies \sim \forall x\{P(x) \vee \exists y[(\sim Q(x, y) \vee Q(y, x)) \vee \forall z(\sim Q(y, z) \vee R(x))]\}$   
 $\implies \exists x \sim \{P(x) \vee \exists y[(\sim Q(x, y) \vee Q(y, x)) \vee \forall z(\sim Q(y, z) \vee R(x))]\}$   
 $\implies \exists x\{\sim P(x) \wedge \sim \exists y[(\sim Q(x, y) \vee Q(y, x)) \vee \forall z(\sim Q(y, z) \vee R(x))]\}$   
 $\implies \exists x\{\sim P(x) \wedge \forall y \sim [(\sim Q(x, y) \vee Q(y, x)) \vee \forall z(\sim Q(y, z) \vee R(x))]\}$   
 $\implies \exists x\{\sim P(x) \wedge \forall y[\sim (\sim Q(x, y) \vee Q(y, x)) \wedge \sim \forall z(\sim Q(y, z) \vee R(x))]\}$   
 $\implies \exists x\{\sim P(x) \wedge \forall y[(Q(x, y) \wedge \sim Q(y, x)) \wedge \exists z \sim (\sim Q(y, z) \vee R(x))]\}$   
 $\implies \exists x\{\sim P(x) \wedge \forall y[(Q(x, y) \wedge \sim Q(y, x)) \wedge \exists z(Q(y, z) \wedge \sim R(x))]\}$   
 $\implies \exists x\forall y\exists z\{\sim P(x) \wedge [(Q(x, y) \wedge \sim Q(y, x)) \wedge (Q(y, z) \wedge \sim R(x))]\}$   
 $\implies$  (答)  $\exists x\forall y\exists z\{\sim P(x) \wedge Q(x, y) \wedge \sim Q(y, x) \wedge Q(y, z) \wedge \sim R(x)\}$
11.  $\forall x\{(P(x) \vee \sim W(x)) \rightarrow \sim \exists y[Q(x, y) \wedge \forall zR(y, z)]\}$   
 $\implies \forall x\{\sim (P(x) \vee \sim W(x)) \vee \sim \exists y[Q(x, y) \wedge \forall zR(y, z)]\}$   
 $\implies \forall x\{(\sim P(x) \wedge W(x)) \vee \forall y \sim [Q(x, y) \wedge \forall zR(y, z)]\}$   
 $\implies \forall x\{(\sim P(x) \wedge W(x)) \vee \forall y[\sim Q(x, y) \vee \sim \forall zR(y, z)]\}$   
 $\implies \forall x\{(\sim P(x) \wedge W(x)) \vee \forall y[\sim Q(x, y) \vee \exists z \sim R(y, z)]\}$   
 $\implies \forall x\forall y\exists z\{(\sim P(x) \wedge W(x)) \vee (\sim Q(x, y) \vee \sim R(y, z))\}$   
 $\implies$  (答)  $\forall x\forall y\exists z\{(\sim P(x) \vee \sim Q(x, y) \vee \sim R(y, z)) \wedge (W(x) \vee \sim Q(x, y) \vee \sim R(y, z))\}$
12.  $\exists x \sim \{(P(x) \vee W(x)) \rightarrow \exists y[\sim Q(x, y) \vee \exists zR(y, z)]\}$   
 $\implies \exists x \sim \{\sim (P(x) \vee W(x)) \vee \exists y[\sim Q(x, y) \vee \exists zR(y, z)]\}$   
 $\implies \exists x\{(P(x) \vee W(x)) \wedge \sim \exists y[\sim Q(x, y) \vee \exists zR(y, z)]\}$   
 $\implies \exists x\{(P(x) \vee W(x)) \wedge \forall y \sim [\sim Q(x, y) \vee \exists zR(y, z)]\}$   
 $\implies \exists x\{(P(x) \vee W(x)) \wedge \forall y[Q(x, y) \wedge \sim \exists zR(y, z)]\}$   
 $\implies \exists x\{(P(x) \vee W(x)) \wedge \forall y[Q(x, y) \wedge \forall z \sim R(y, z)]\}$   
 $\implies$  (答)  $\exists x\forall y\forall z\{(P(x) \vee W(x)) \wedge Q(x, y) \wedge \sim R(y, z)\}$